Solutions to Problems

Problem 5.1

$$\sin 210^\circ = \sin(180^\circ + 30^\circ) = \cos 180^\circ \sin 30^\circ = -0.5.$$

$$\cos 120^\circ = \cos(90^\circ + 30^\circ) = -\sin 30^\circ = -0.5.$$

$$\tan 330^\circ = \frac{\sin 330^\circ}{\cos 330^\circ} = \frac{\sin(360^\circ - 30^\circ)}{\cos(360^\circ - 30^\circ)} = -\frac{\sin 30^\circ}{\cos 30^\circ} = -\frac{1}{\sqrt{3}}.$$

Problem 5.2

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin(\theta/2 + \theta/2)}{\cos(\theta/2 + \theta/2)}$$

$$= \frac{2\sin(\theta/2)\cos(\theta/2)}{\cos^2(\theta/2) - \sin^2(\theta/2)} = \frac{2\tan(\theta/2)}{1 - \tan^2(\theta/2)}.$$

Problem 5.3

In Figure B.1, $a = d + f$ and

$$\frac{d+f}{\sin\alpha} = \frac{b}{\sin\beta}. \qquad (B.1)$$

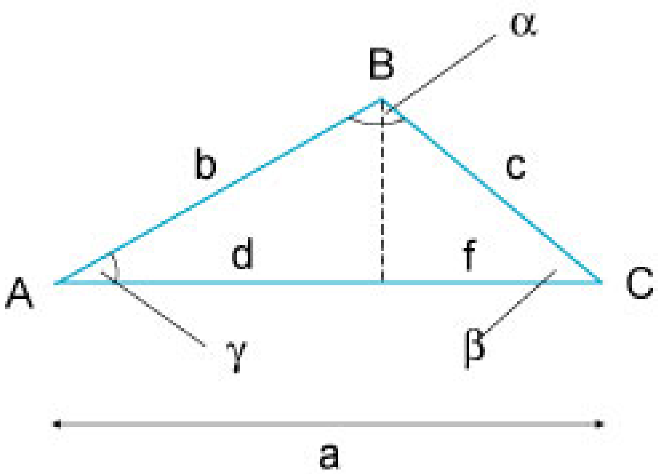

Figure B.1

From the right-angled triangle formed by dropping a line from the point B to meet the line AC at right angles, $d = b\cos\gamma$, and $f = c\cos\beta$. Substitution in equation B.1 gives

$$\frac{b\cos\gamma + c\cos\beta}{\sin\alpha} = \frac{b}{\sin\beta}$$

and

$$\begin{aligned}\sin\alpha &= \frac{\sin\beta(b\cos\gamma + c\cos\beta)}{b}\\ &= \sin\beta\cos\gamma + \frac{c}{b}\sin\beta\cos\beta\\ &= \sin\beta\cos\gamma + \left(\frac{\sin\gamma}{\sin\beta}\right)\sin\beta\cos\beta\\ &= \sin\beta\cos\gamma + \sin\gamma\cos\beta.\end{aligned}$$

But $\beta + \gamma = \pi - \alpha$ and hence $\sin(\beta+\gamma) = \sin\alpha$ and finally

$$\sin(\beta+\gamma) = \sin\beta\cos\gamma + \sin\gamma\cos\beta.$$

Problem 5.4

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\sin\theta\cos\theta}{1+\sin\theta}\right) = \frac{(1+\sin\theta)(\cos^2\theta - \sin^2\theta) - \sin\theta\cos^2\theta}{(1+\sin\theta)^2},$$

from equation 2.11,

$$= \frac{\cos^2\theta - \sin^2\theta + \sin\theta\cos^2\theta - \sin^2\theta = \sin\theta\cos^2\theta}{(1+\sin\theta)^2}$$

$$= \frac{\cos^2\theta - \sin^2\theta + \sin\theta\cos^2\theta - \sin\theta(\sin^2\theta + \cos^2\theta)}{(1+\sin\theta)^2}$$

$$= \frac{\cos^2\theta - \sin^2\theta + \sin\theta(\cos^2\theta - 1)}{(1+\sin\theta)^2} = \frac{\cos^2\theta - \sin^2\theta - \sin^3\theta}{(1+\sin\theta)^2}.$$

Problem 5.5

$$\int \sin^2\theta\cos^3\theta\mathrm{d}\theta = \int \sin^2\theta\cos^2\theta\mathrm{d}(\sin\theta)$$

$$= \int \sin^2\theta(1-\sin^2\theta)\mathrm{d}(\sin\theta) = \int(\sin^2\theta - \sin^4\theta)\mathrm{d}(\sin\theta)$$

$$= \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta = \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^3\theta(1-\cos^2\theta)$$

$$= \frac{2}{15}\sin^3\theta + \frac{1}{5}\sin^3\theta\cos^2\theta.$$

Problem 5.6

From equations 5.18 and 5.19

$$\cos\theta\cos\phi = \frac{1}{4}(e^{j\theta}+e^{-j\theta})(e^{j\phi}+e^{-j\phi})$$

$$= \frac{1}{4}(e^{j(\theta+\phi)}+e^{-j(\theta+\phi)}+e^{j(\theta-\phi)}+e^{-j(\theta-\phi)}). \qquad (B.2)$$

$$\sin\theta\sin\phi = -\frac{1}{4}(e^{j\theta}-e^{-j\theta})(e^{j\phi}-e^{-j\phi})$$

$$= -\frac{1}{4}(e^{j(\theta+\phi)}+e^{-j(\theta+\phi)}-e^{-j(\theta-\phi)}-e^{j(\theta-\phi)}). \qquad (B.3)$$

Addition of equations B.2 and B.3 gives

$$\cos\theta\cos\phi+\sin\theta\sin\phi = \cos(\theta-\phi).$$

Problem 5.7

$$2\sin\frac{(\theta+\phi)}{2}\cos\frac{(\theta-\phi)}{2}$$

$$= \left(2\sin\frac{\theta}{2}\cos\frac{\phi}{2}+2\sin\frac{\phi}{2}\cos\frac{\theta}{2}\right)\times\left(\cos\frac{\theta}{2}\cos\frac{\phi}{2}+\sin\frac{\theta}{2}\sin\frac{\phi}{2}\right).$$

Multiplying out the brackets and using equation 5.7 gives

$$2\sin\frac{(\theta+\phi)}{2}\cos\frac{(\theta-\phi)}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\sin\frac{\phi}{2}\cos\frac{\phi}{2}$$

$$= \sin\theta+\sin\phi.$$

Problem 5.8

$$\frac{(2-3j)(1+2j)}{4+3j} = \frac{8+j}{4+3j} = \frac{(8+j)(4-3j)}{\sqrt{4^2+3^2}} = \frac{7-4j}{5}$$

Expressing z in polar form, the modulus

$$r = \sqrt{(7/5)^2+(4/5)^2} = \frac{1}{5}\sqrt{65}$$

and the angle θ is given by

$$\tan\theta = -\frac{4/5}{7/5} = -4/7.$$