

Section 3: RELATIVITY AND QUANTUM MECHANICS

In this section, we summarize the essential results of Special Relativity and Quantum Mechanics that are particularly relevant to the physics of atoms and nuclei. . Both are vast subjects and results will be presented without proofs.

Relativity

Here, we consider only **Special Relativity**, which deals with effects of **uniform relative motion** between two observers on measurements in physics.

Special relativity is based on two fundamental hypotheses. These are:

Hypothesis I: **Absolute uniform motion is undetectable.**

- Air travellers are unaware of a plane's motion (in steady flight).
- We are unaware of earth's speed as it orbits the sun or of the solar system is moving at enormous speed relative to the galaxy.

Fundamental consequence of Hypothesis I:

The **laws of physics are the same (invariant) irrespective of any uniform motion.**

- There is **no** physical measurement, which can detect uniform motion.
- Hence, as an observer, **we can always choose to assume that we are at rest** (this greatly simplifies calculations).

Hypothesis II: **In vacuum, light has the same speed c for all observers.**

This arose from attempts to detect the earth's motion of relative to the **ether**:

- **Light** behaves like a **wave** and the **ether** was the medium on which the waves travelled - like **sound waves in air** or **ripples** on the surface of **water**.
- The speed of sound waves or water waves changes relative to us, depending on our own speed through the water or air. Therefore, the speed of light should change relative to the direction of the earth's motion about the sun.
- Many sensitive measurements made. The most famous was carried out by the American physicists Michelson and Morely. In none of the experiments was any effect detected and we assert that:

c is a fundamental constant of nature.

Now, electromagnetic waves and the speed of light can be derived directly from Maxwell's laws of electricity and magnetism, i.e. **the speed of light is a result of the laws of physics**, which are unchanged by any assumed uniform motion.

Thus, the **constancy of c a necessary consequence of Hypothesis I!**

Consequences of special relativity:

There are important consequences of special relativity that alter the way we consider basic physical quantities like

time space mass energy:

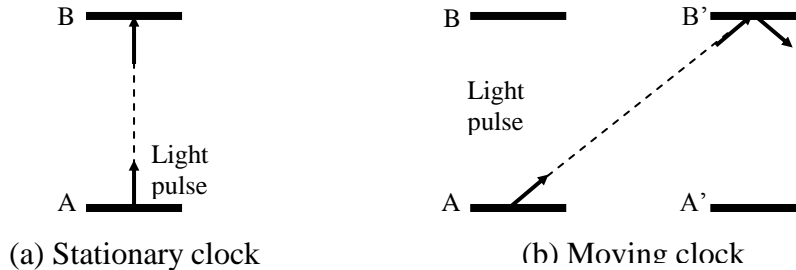
We will now consider some of these:

Relative to an observer:

1. A moving clock runs more slowly than when it is at rest.

Imagine a special clock consisting of two parallel mirrors with a pulse of light being reflected back and forth between them.

Time interval = time between successive reflections
= time for light to travel between the mirrors.



For a **stationary** clock (relative to the observer): Distance travelled by light pulse = AB

$$\text{Time interval } t_0 = AB/c$$

For a **moving** clock (with speed v in a direction perpendicular to the line joining the two mirrors);

Distance travelled by the light pulse = AB'

$$\text{Time interval } t = AB'/c$$

Clearly, $AB' > AB$ so $t > t_0$

From Pythagoras: $(AB')^2 = (AA')^2 + (AB)^2$, whence

$$\frac{AB}{AB'} = \sqrt{1 - \left(\frac{AA'}{AB'}\right)^2} = \sqrt{1 - \beta^2}$$

where $\beta = v/c$.

Thus, if the speed of light is constant and the same for all observers, the time between reflections for the moving clock is increased by the factor

$$\gamma = AB'/AB = 1/\sqrt{1-\beta^2}$$

i.e. a moving clock apparently runs more slowly.

Now, if a moving **light** clock runs more slowly, **any** moving clock will also run more slowly. This follows because any observer can compare two clocks, which are side by side and must necessarily come to the same conclusion about the timekeeping of both of them.

2. A rod moving parallel to its length is shorter than when it is at rest.

E.g. A train appears **shorter** to an observer standing by the track compared with the length measured by an observer travelling on the train. - link between **space** and **time**

3. The mass of a moving object is greater than when it is at rest.

- c is an upper limit to an object's speed v
- So, as $v \rightarrow c$, continued application of a force achieves diminishing returns, i.e. **acceleration decreases**.

- Thus, (inertial) mass = force/acceleration, **increases** as v increases.

We define **rest mass** as the mass of an object at rest relative to an observer.

Adding two speeds v_1 and v_2 :

Relativistic result is: $\frac{(v_1 + v_2)}{(1 + v_1 v_2/c^2)}$ (N.B. **not** $v_1 + v_2$)

Consistency check: substitute c for either v_1 or v_2 - the result is always c .

4. Energy is related to mass: $E = mc^2$

- As work is done on an object (with $v \sim c$), mass m increases: - Therefore, mc^2 increases.
- In fact, the **increase = the work done** → confirming **energy conservation**.

Relativity and light.

- Light has **energy** and, therefore, **effective mass** ($m = E/c^2$). E.g. light is bent in a gravitational field.
- Light also has speed (c) and, therefore, **momentum** $p = mc = (E/c^2) \times c = E/c$.
- However, light is **special** because it cannot be at rest ($c = \text{constant}$).
- If light loses **energy**, its speed does not change but its **frequency decreases**.
- This is another very important result, which leads to a **quantum-mechanical** description.
- The constancy of c means that the **rest mass of light is zero**.

Particles with rest mass.

- If rest mass $m_0 > 0$, there is **rest energy** $= m_0 c^2$.
- Rest energy is not normally available, but an exception is **antimatter**:

Antimatter is a lot like ordinary matter, except we must treat the **sign** of its rest mass to be opposite to that of ordinary matter.

Antimatter is **unstable** in the presence of matter \rightarrow an electron and an anti-electron (positron) will **annihilate** resulting in pure electromagnetic energy (radiation) with zero rest mass.

- The total energy of a moving mass = rest energy + kinetic energy (KE).
- Therefore, effective mass: $m = m_0 + \text{KE}/c^2$, i.e. a moving object gets more massive.
- The increase is by the same factor γ as that for the time interval of a moving clock relative to a stationary one. $m = \gamma m_0 = m_0 / \sqrt{(1 - \beta^2)}$ where $\beta = v/c$.

For **low speeds** (relative to c), i.e. $\beta \ll 1$, total energy $= mc^2 = m_0 c^2 (1 + \frac{1}{2} \beta^2 + \dots)$

So, taking only the first term in the expansion: $\text{KE} \approx \frac{1}{2} m_0 c^2 \beta^2 = \frac{1}{2} m_0 v^2 \ll m_0 c^2$.

Other forms of energy

Energy appears in many different forms, **all manifest as mass**.

- A spring: mass(compressed) > mass(uncompressed)
- Heated object: mass(hot) > mass(cold)
- Potential energy: mass(earth + moon) < mass(earth) + mass(moon)

The last example is very important for atoms and nuclei

The mass of a system depends on the **internal (binding) energy**.

Binding energy is negative:

e.g. To ionise an atom, there is a need to *input* energy (E) to overcome the binding energy. Therefore, the sum of the separate masses (ion + electron) $>$ mass of the atom by E/c^2 .

- Similarly, neutrons and protons (nucleons) are bound in nuclei by a strong nuclear force. In fact, the mass of a nucleus is significantly less than the masses of its constituent nucleons
→ **NUCLEAR ENERGY**

Quantum mechanics

Quantum mechanics was developed as a need to reconcile a paradox that radiation (e.g. light, electron beams etc.) exhibits both **wave-like** and **particle-like** properties.

These two concepts are completely different:

- A **particle** has no extent and a definite location at any time.
- A **wave** has extent, and does not have a well-defined location at any time.

Light is our most familiar illustrative example.

Wave aspects of light.

Light (electromagnetic radiation) is classified according to **wavelength** or **frequency**.

Light propagates as a **wave** - which explains interference and diffraction when light is passed through narrow slits or a diffraction grating.

In the case of **radio waves**, we can measure both **wavelength** and **frequency**.

Classical theory of electromagnetic radiation **predicts** em waves and is very successful in that it explains: reflection, refraction, polarization, interference etc.

The **wave nature of light** is **clear and irrefutable**.

Particle aspects of light

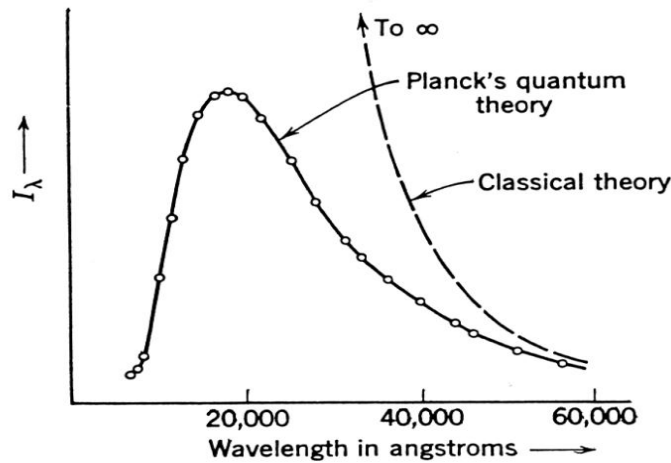
- Light has **mass, speed** and **momentum**.
- Light is emitted and detected in **discrete amounts** of energy, called **energy quanta**.
- A 'particle' of light is called a **photon**. It interacts and is detected as a whole unit.
- Example: Compton scattering of light (em radiation) by electrons (see later). The energy and momentum of a recoiling electron is consistent with it having been struck by a photon with energy E (proportional to frequency) and momentum ($= E/c$).
- The photon theory also explains the **photoelectric effect** (Einstein 1905).

Dual nature of light

Wave and particle properties are related:

- Photon energy is proportional to frequency ν : $E = h\nu$ known as the Planck equation, where $h = \text{Planck's constant.}(6.626 \times 10^{-34} \text{ J s})$.

This idea resolved the problem of **black-body radiation** from a hot object: (Planck 1900).



- Classical theory predicts the **emission probability** of radiation to be proportional to N , the **number of ways** radiation of a particular wavelength can be trapped in an enclosure.
 - Now, N increases with frequency (decreasing wavelength) without limit.
 - At low frequencies, (high wavelength) the observed emission approximates classical theory but deviates progressively at higher frequency (see above).
 - However, with an additional constraint that radiation must be emitted in **discrete** energy units proportional to frequency, the **probability** of obtaining a concentration of energy to emit a unit **decreases** with increasing frequency (decreasing wavelength) - as observed.

Momentum is related to wavelength:

$$p = E/c = h\nu/c = h/\lambda, \quad \text{i.e. } \boxed{p = h/\lambda}$$

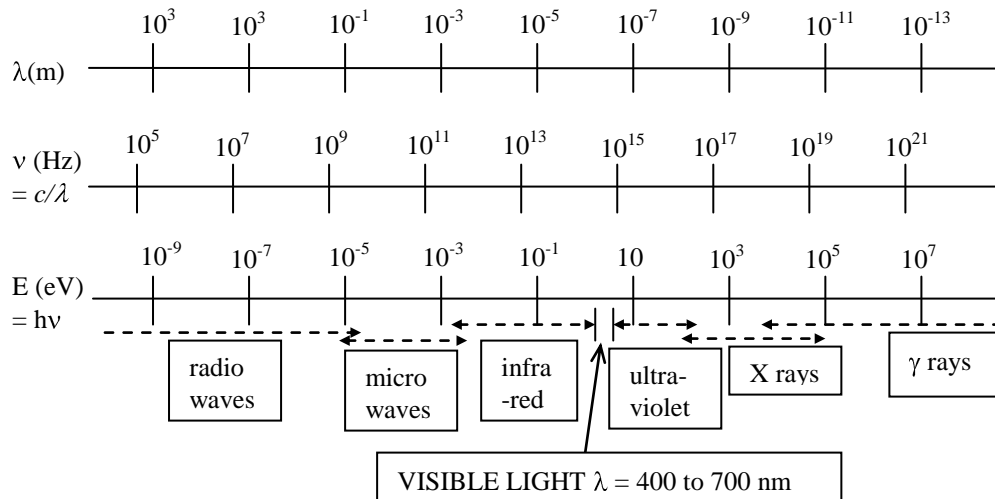
This is known as the **de Broglie** relation. Evidence for it is obtained from measurements of Compton scattering - referred to above.

The electromagnetic spectrum

- The spectrum is subdivided into **types**, each covering a certain range of wavelength λ , frequency ν or energy E .

- The boundaries between the different ranges are approximate and often overlap.

e.g. X-rays and γ -rays



- X-rays normally refer to photons emitted from heavy **atoms**.
- γ rays normally refer to photons emitted by **nuclei**.
- X-ray photon energies can exceed γ -ray energies.
- Visible light is interpreted by the human eye/brain as **colour**.
- Longest visible wavelength is red and the shortest is violet.
- Wavelength of infra-red > red light. Wavelength of ultra-violet < violet light.

Basic quantum mechanics

The relations $E = h\nu$ and $p = h/\lambda$, are completely general.

Any particle has wave properties

i.e. Atomic constituents: electrons, protons and neutrons, exhibit **wave-like properties**.

- They have an associated **de Broglie wavelength**, $\lambda = h/p$. Experiments show that beams of electrons or neutrons do exhibit **diffraction** effects - like em radiation of the same λ .

However, the wave nature of a particle is only significant if $\lambda \approx$ typical dimension of the situation:

Example 1. Electron orbiting an atom - kinetic energy $E = 10$ eV.

Momentum $p = \sqrt{2mE}$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34} \text{ J s}}{\sqrt{2 \times 9.1 \times 10^{-31} \text{ kg} \times 10 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV}}} = 3.9 \times 10^{-10} \text{ m}$$

This is comparable with atomic size, therefore, its **wave properties are important**.

Example 2. A 1 tonne car travelling at 30 mph (13.3 m s^{-1}).

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J s}}{10^3 \text{ kg} \times 13.3 \text{ m s}^{-1}} = 5 \times 10^{-38} \text{ m}.$$

\approx nuclear diameter $\div 10^{23}$!!! In this case, wave properties are **completely negligible**.

An important result of quantum mechanics for atoms and nuclei is that

- A confined object is like a trapped wave.
- Its **wavelength** is determined by the size of the confining region. Thus, there is an associated **momentum** and, therefore, **energy** (sometimes called zero-point energy):

i.e. **It is impossible for a confined object to be completely at rest.**

However, h is very small and the object has to have a very small mass for this quantum energy to be significant – e.g. in atoms and nuclei.

The **quantum wave** measures the **probability of a particle's existence in space**. i.e. where it is likely to be found. The more confined the particle, the greater is its wave amplitude and the greater is the **probability** that it is to be found at some point in the confining region.

It also follows that:

- The **more confined** the particle, the **greater** is its **zero-point energy**.
- This explains why an atomic electron does not radiate energy and spiral in to the nucleus under the force of attraction between them.

According to quantum mechanics, an electron in an atom exists at an **energy minimum**, which is where there is a balance between quantum zero-point (kinetic) energy trying to **increase** the orbit size and electrical attraction trying to **decrease** it.

This idea forms the basis of the way electrons are arranged in atoms.