Section 7: BASIC NUCLEAR CONCEPTS

In this section, we present a basic description of atomic nuclei, the stored energy contained within them, their occurrence and stability

Basic Nuclear Concepts

EARLY DISCOVERIES [see also Section 2]

Radioactivity - discovered in 1896 by Henri Becquerel.

Types of radiation observed:

alpha (α) rays (^{4}He nuclei); beta (β) rays (electrons) ; gamma (γ) rays (photons)

Proposed atomic models: built of positively and negatively charged components.

- Planetary model: Light electrons (-charge) orbiting a massive nucleus (+charge):

- 'Plum pudding' model (J. J. Thompson): In this model, electrons are embedded but free to move in an extended region of positive charge filling the entire volume of the atom. Thompson found it difficult to develop this model. For example he could not account for the patterns of discrete wavelengths in light emitted from excited atoms.

In the early 1900s, Rutherford and co-workers, by performing experiments scattering α particles off gold, confirmed the **planetary model** with a small, massive nucleus at its centre. The problem of the stability of such an atom was realized early on but not explained until the development of quantum mechanics [see Section 3].

Discovery of the neutron 1932 – The neutron was identified by James Chadwick from observations of the effects of radiation emitted when beryllium is bombarded with alpha particles. This gave the basic nuclear framework (Heisenberg, Majorana and Wigner) that the nucleus consists of **nucleons** (neutrons and protons) held together by a strong, short-range binding force, with a strength independent of the type of nucleon.

Nuclear size and density

Scattering experiments showed that the nuclear radius varies as cube root of the mass number A, i.e.

$$r = r_0 A^{1/3} \tag{5.1}$$

with $r_0 \approx 1.2\times10^{-15}$ m = 1.2 fm.

Example: Carbon, $A = 12$; we get $r_{\text{nuc}} = 2.7\times10^{-15}$ m. From Section 1, the radius of a carbon atom = 1.3×10^{-10} m. Therefore, the ratio: $\dfrac{r_{\text{nuc}}}{r_{\text{atom}}} = \dfrac{2.7\times10^{-15}}{1.3\times10^{-10}} \approx \dfrac{1}{50{,}000}$.

The nucleus is a **tiny** object compared with an atom.

The nuclear volume ($4\pi r^3/3$) $\propto$ A, which implies **nuclear matter density ~ constant**. Also, nuclear mass ~99.98% of the atomic mass, from which we deduce that the density of nuclear matter is **huge** ~10^{18} kg m^{-3} $\Rightarrow 1\ \text{cm}^3 \approx$ 10,000 battleships!

NUCLEAR MASS AND ENERGY

Binding energy

Nuclear mass < sum of the masses of constituent neutrons and protons.

E.g. Deuteron (bound neutron and proton):

$$m_d = 2.01355\ \text{u} < m_n + m_p = 1.00867 + 1.00728 = 2.01595\ \text{u}$$

Difference in mass $\Delta m = m_d - (m_n + m_p)$.

$E = mc^2$ (Einstein) enables us to express energies in either mass or energy units.

Thus, $\Delta m\, c^2$ = the binding energy of nucleons in a nucleus, which is negative. Its magnitude is the energy released when the nucleons fuse into a nucleus. Conversely, it is the energy needed to separate the nucleus into its N neutrons and Z protons.

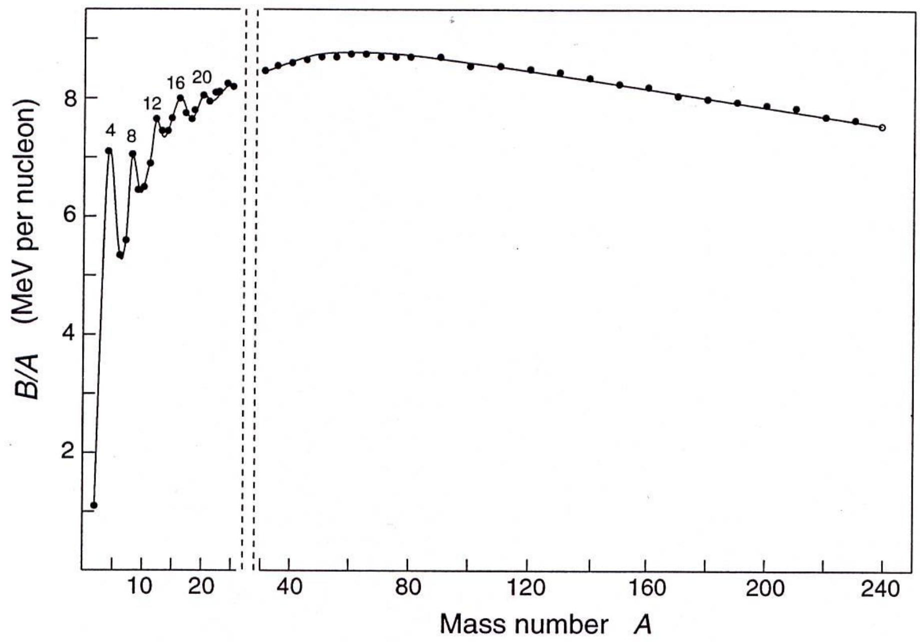
Released energy or nuclear **binding energy** B is given by

$$B(A,Z) = (Zm_p + Nm_n - m(A,Z))\, c^2 = \Delta mc^2 \qquad (5.2)$$

where m_p, m_n and $m(A,Z)$ are masses of the proton, neutron and nucleus (atomic mass A), respectively.

B increases with A – and we usually quote the average **binding energy per nucleon** (B/A).

This figure shows the variation of B/A with A:



- At low A, B/A increases with A to a broad maximum near $A = 60$ of about 8.6 MeV per nucleon.
- Beyond $A = 60$, there is a gradual decrease to about 7.6 MeV per nucleon for the heaviest nuclei.
- Nuclei with A greater than 238 are not found in significant quantities in the earth's crust.
- Several sharp peaks below $A = 30$ correspond to nuclei ^{4}He, ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne and ^{24}Mg. The ^{4}He nucleus (α particle) is particularly stable and the A and Z of the other nuclei are multiples of the α particle. Their extra stability is taken as evidence that their structure resembles that of a collection of α particles.
- The approximate constancy of B/A over most of the range is indicative of the **saturation property of the nuclear force - (**see later).

Q value

Q value = energy change in any transformation process, e.g. a nuclear reaction or a radioactive decay. From energy conservation: $Q = (m_\mathrm{i} - m_\mathrm{f})c^2$ (5.3)

The process is **endothermic** or **exothermic** depending on whether the Q value is **negative** or **positive**.

Example: Radioactive decay of ^{238}U: $^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He}$

$Q = [m(^{238}\mathrm{U}) - m(^{234}\mathrm{Th}) - m(^{4}\mathrm{He})]\, c^2 = (238.050783 - 234.043596 - 4.002603)\, c^2 = 4.268$ MeV, using 1 u ≡ 931.49 MeV.

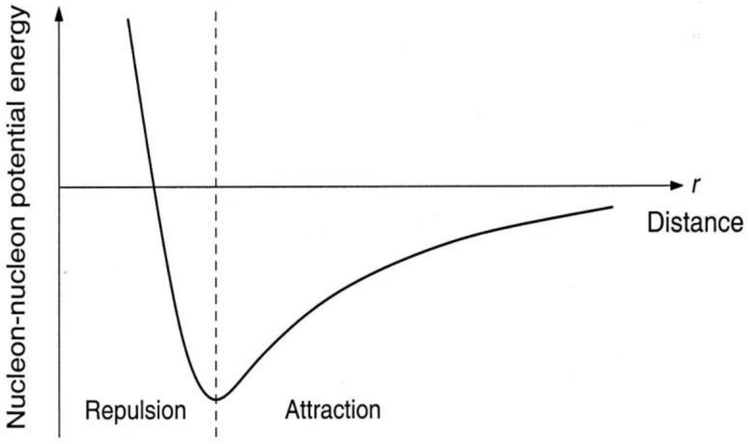
Q value is **positive** (as it must be for the decay to occur at all) and appears as kinetic energies of ^{234}Th and ^{4}He.

Nuclear force

About $\times 10^{40}$ stronger than the gravitational force.

About 50 times stronger than the electric force between two protons.

- It is **short ranged** - falls to zero at inter-nucleon distances greater than a few fm.
- It is **charge independent -** same between n-n, p-p, or n-p.
- At distances >few fm, the nuclear force is attractive.
- At very close distances (<1 fm), the force becomes **repulsive**.
- Nucleons tend to maintain an **equilibrium distance** where the repulsion and attraction are balanced.
- This equilibrium distance corresponds to a **minimum** in nuclear potential energy versus nucleon separation (see below).



- The constant inter-nucleon separation explains the approximately constant density of nuclear matter.
- The **short range** of the nuclear force means a nucleon senses only its nearest neighbours.
- In a light nucleus, **all** the nucleons will sense each other – so the binding energy of the nucleon (*B/A*) increases with *A*.
- In heavy nuclei, nuclear size > range of nuclear force. A nucleon senses approximately a constant number of neighbours and hence, the nuclear *B/A* levels off at high *A*. This feature is referred to as the **saturation of the nuclear force**.

Coulomb force

This is a repulsive force acting between protons inside the nucleus. It is weaker than the nuclear force. At a typical inter-nucleon separation, the Coulomb energy of two protons is about 1/50th of the nuclear energy.

However, the Coulomb force is **long-ranged:** A proton feels the electric force due to all the other protons. Thus, while the nuclear *B/A* levels off as *A* and the nuclear size increase, the Coulomb energy continues to increase because *Z* increases approximately as *A/*2.

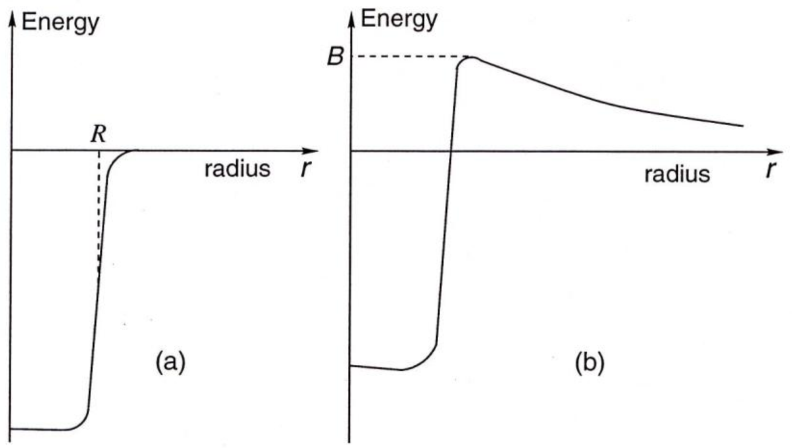
The Coulomb energy is of opposite sign to the nuclear energy, hence *B/A* gradually falls off at large *A* from a maximum. at $A \approx 60$, as shown in the figure.

NUCLEAR POTENTIAL AND ENERGY LEVELS

Nuclear potential

Outside the nucleus $r > R$, (nuclear radius): the nuclear PE experienced by a neutron → 0 in a few fm.

Inside the nucleus, any nucleon feels an **average potential** due to the interaction with its neighbours.



neutron proton

In addition, a proton feels the repulsive Coulomb potential, which decreases the net interior PE. It also gives rise to a (Coulomb) barrier in the surface region, which means that energy is required to push a proton towards the surface of a nucleus. This energy increases up to the edge of the nucleus where the total PE reaches a peak *B* as the nuclear attraction begins to take effect. Beyond the peak, the nuclear force dominates and the PE decreases at closer distances to form a **potential well**.

Nucleon states

Nucleons of both types bound in a nucleus are confined in a potential-energy well**.** These nucleons can occupy only **allowed states** – like electrons in an atom. However, energies of nuclear states (**millions** of eV) are much greater than electron atomic energies (eV).

We can estimate the nucleon kinetic energy, Using the de Broglie relation: $p = h/\lambda$
Inside a nucleus, the nucleon wavelength $\lambda \sim$ nuclear size $D \approx 10$ fm.

Therefore, $p = h/D$ and the kinetic energy:

$$\mathrm{KE} = \frac{p^2}{2m} = \frac{h^2c^2}{2mc^2D^2} \tag{5.4}$$

where m = nucleon mass.

Substituting values gives $$\mathrm{KE} = \frac{h^2c^2}{2mc^2D^2} = \frac{1238^2\,(\mathrm{MeV\,fm})^2}{2\times 931(\mathrm{MeV})\times 100(\mathrm{fm})^2} \approx 8\,\mathrm{MeV}.$$

This energy must be overcome by the nuclear PE to bind a nucleon in a nucleus.

Inside a nucleus, nucleons arrange themselves in shells (like electrons in an atom). They obey the Pauli Exclusion Principle that no two identical nucleons can exist in the same state.

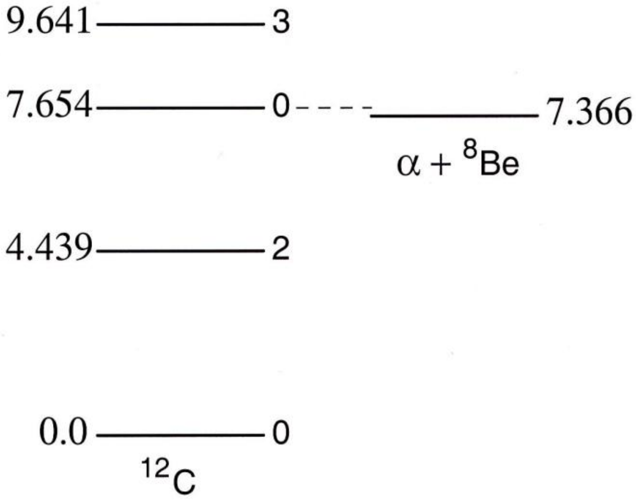
Nuclear energy levels

In the **ground state** of a nucleus, neutrons and protons occupy their lowest states.

In an **excited state,** one or more nucleons are promoted to higher-energy allowed states - analogous to exciting an atom by promoting electrons to higher-energy states.

At low excitation, nuclear levels occur at discrete energies. Each nuclide has its own characteristic energy spectrum.

Example: ^{12}C:



The integer labels the angular momentum quantum number, which specifies the angular momentum (spin) in units of $h/2\pi$.

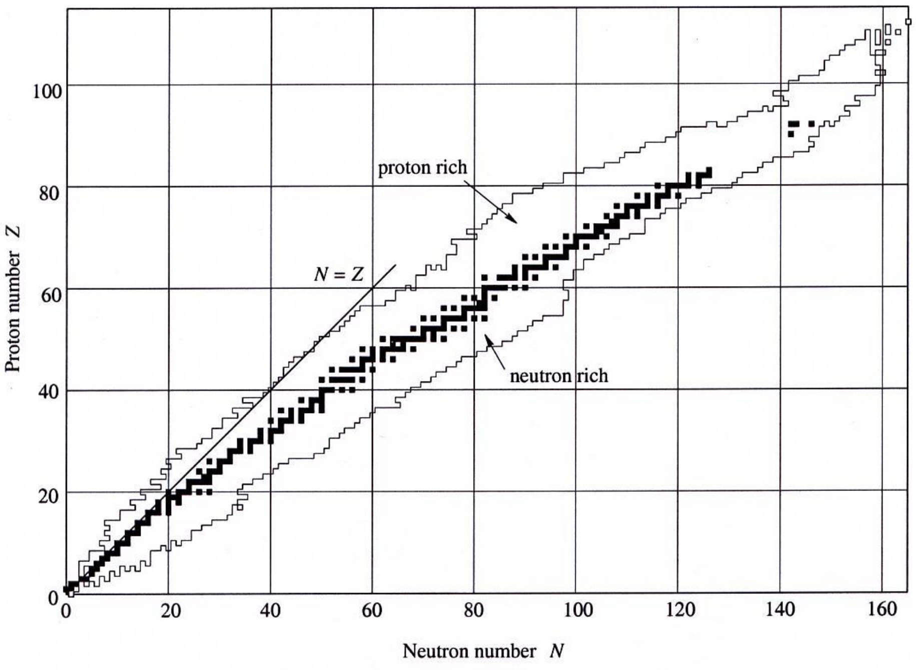
A nucleus in an excited state decays (de-excites) after a short time. Often it decays by emitting a γ ray, while undergoing a transition to a lower energy level. However, if the excitation energy is high enough, the nucleus may decay by emitting a particle such as a neutron or an α particle - e.g. the 7.654 MeV state in ^{12}C can α decay into α+^{8}Be.

OCCURRENCE AND STABILITY OF NUCLEI

Chart of the nuclides [see figure below]

Known stable nuclei and very long-lived unstable nuclei (black squares) follow a particular line known as the line or **valley of stability**.

At low A, $N \approx Z$. This is because, when the Coulomb PE is weak (low Z), neutrons and protons occupy allowed states independently of each other and approximately equally.



However, at higher *A*, we find $N > Z$. The Coulomb energy becomes more and more important at high *Z*, making the potential well shallower for protons. Their allowed states are shifted upwards in energy compared with the neutron states, which means that the lowest energy is when $N > Z$.

NUCLEAR INSTABILITY

Either side of the Valley of Stability:

For a given *A*, nuclei for which *N* and *Z* lie above or below the line of stability, are not the most stable. Energy is released by changing a neutron into a proton or vice versa. The force, causing such a change, is the **weak nuclear force**, whose strength is about 10^{-12} times that of the strong nuclear force and so it plays no part in holding the nucleus together.

It causes a **neutron-rich** or **proton-rich** nucleus to decay into a more stable form via **β radioactivity –** see Section 8.

Beyond the Valley of Stability:

There are upper limits to *A* and *Z* for which stable nuclei can exist (A = 209 and Z = 83).

Beyond ^{209}Bi, nuclei may transform into a more stable product by emitting an α particle - via **α radioactivity**. Naturally occurring thorium and uranium undergo alpha decay, albeit with very long average lifetimes.

Very heavy nuclei can also decay by undergoing **fission** into two approximately equal fragments – see Section 8.