Section 8: RADIOACTIVE DECAY

In this section, we describe radioactivity - how unstable nuclei can decay - and the laws governing radioactive decay.

Radioactive Decay

Naturally occurring radioactive nuclei undergo a combination of α, β and γ emission. Artificially produced nuclei may also decay by spontaneous fission, neutron emission and even proton and heavy-ion emission.

Any decay process is subject to the same basic law.

RADIOACTIVE DECAY LAW

The rate of decay (number of disintegrations per unit time) is proportional to N, the number of radioactive nuclei in the sample $\quad dN/dt \propto -\lambda N \quad$ (6.1)

The negative sign signifies that N is **decreasing** with time. λ is called the **decay constant** - probability per unit time that a given radioactive nucleus will decay.

- Large $\lambda \Rightarrow$ rapid decay; small $\lambda \Rightarrow$ slow decay.

Equation (6.1) can be integrated to give $\quad N(t) = N_0 \exp(-\lambda t) \quad$ (6.2)

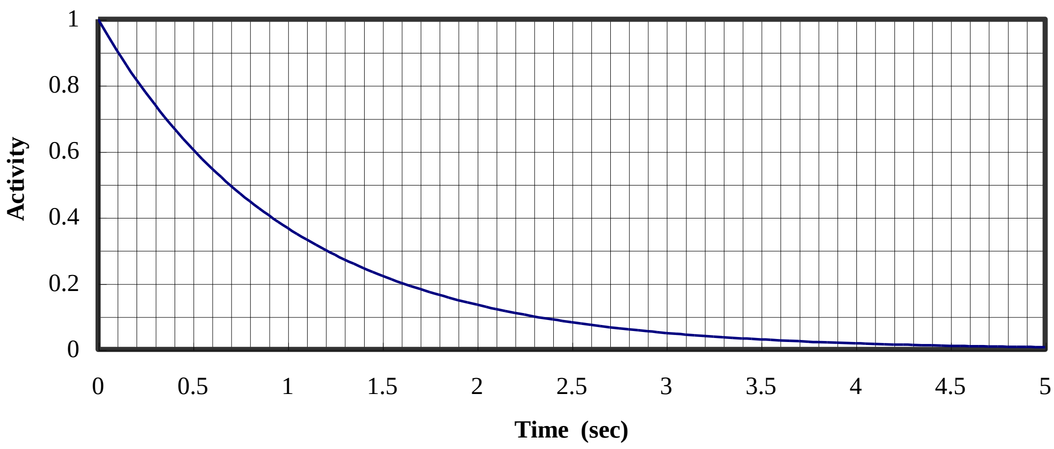
where N_0 = number of radioactive nuclei at $t = 0$.

ACTIVITY AND HALF-LIFE

Activity: Number of disintegrations per unit time:

$$A(t) = \lambda N(t) = \lambda N_0 \exp(-\lambda t) = A_0 \exp(-\lambda t) \quad (6.3)$$

This has the same exponential fall off with time as $N(t)$.



Half-life: Time for half the radioactive nuclei in the sample to decay.

Substituting $N_0 = N_0/2$ and $t = t_{1/2}$ into Eq. (6.2) gives $t_{1/2} = \ln 2/\lambda$ (6.4)

The figure above shows the activity of a sample decaying at a rate of exp(-t). The half-life of this sample = ln2 (≈0.7 s).

DECAY CHAINS

When nuclei A decay into **stable** nuclei B, the number of each present at time t is

$$N_A(t) = N_A(0)e^{-\lambda t} \quad \text{and} \quad N_B(t) = N_A(0)(1 - e^{-\lambda t}) \qquad (6.5)$$

where only nuclei A are present initially.

The number of nuclei A (**parent** nuclei) decreases with time.

The number of nuclei B (**daughter** nuclei) increases from zero, initially, and approaches $N_A(0)$ as $t \to \infty$, i.e. all the parent nuclei eventually become daughter nuclei.

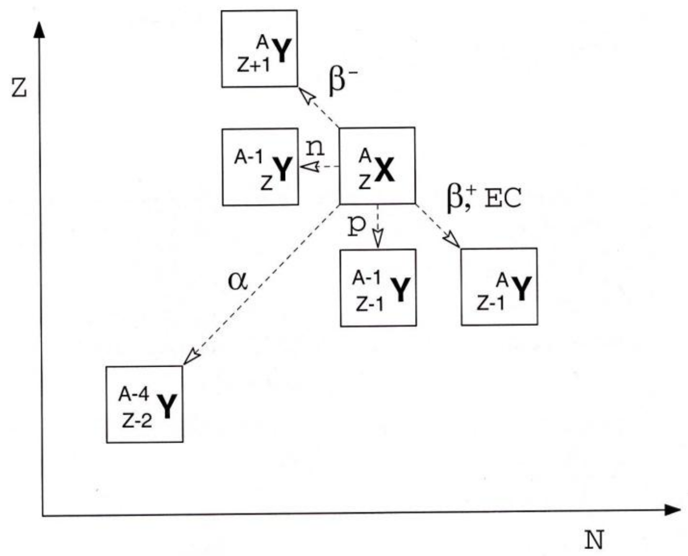
The total number of nuclei is constant: $N_A(t) + N_B(t) = N_A(0)$.

If nuclei B are also radioactive, the above equations do not apply, since, as nuclei B are produced, they also decay. The daughter nuclei of B may also be radioactive and a **decay chain** is set up: A → B → C → ···· etc.

We shall give a number of examples of decay chains in the next section.

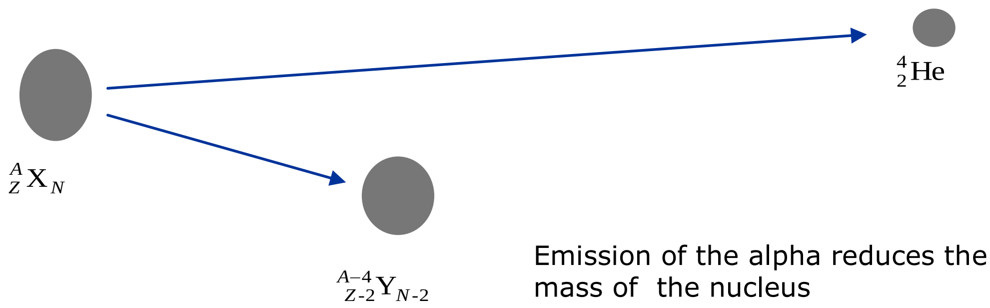
TYPES OF RADIOACTIVE DECAY

The figure below shows how a variety of decay mechanisms transform an initial (parent) nucleus $^{A}_{Z}\mathrm{X}_N$ into different final (daughter) nuclei.



Here we shall consider a number of basic decay mechanisms. They are: alpha decay; beta decay (β^-, β^+ and electron capture); gamma decay and internal conversion; spontaneous fission.

ALPHA DECAY



The parent nucleus $^{A}_{Z}X_{N}$ emits an α particle. The daughter product is 4 mass units lighter and with 2 fewer units of electric charge: $^{A-4}_{Z-2}X_{N-2}$.

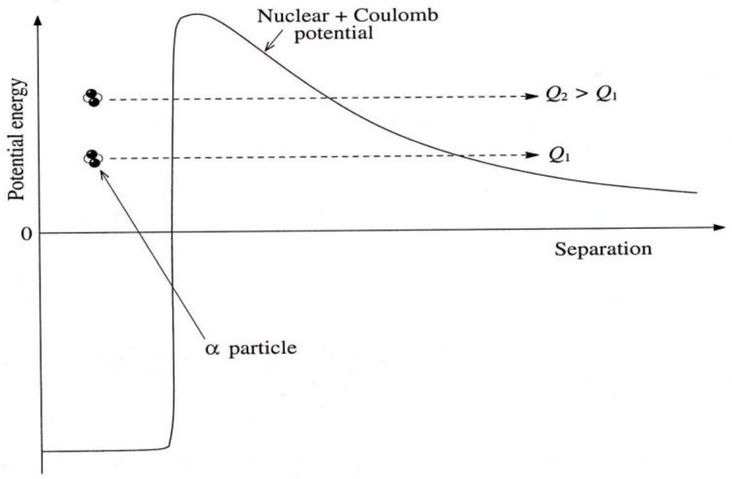
Characteristics of alpha decay

If it is energetically favourable for a nucleus to lose mass, the α particle (charge +2*e* and mass 4.0026 u ≡ 3.7 GeV) is most commonly emitted because it is a tightly bound system.

Alpha decay occurs almost exclusively in heavy nuclei because *B/A* **decreases** with *A* when *A* is large and so the daughter product of α decay is **more stable** than the parent. Mass numbers of nearly all α emitters > 209 and typical α-particle kinetic energies E_α = 4 to 6 MeV. Alpha particle energies are well-defined.

- *As* α particles pass through matter, they lose energy rapidly. Alpha particles of a few MeV are easily stopped by paper or skin – e.g. the range of a 5 MeV α is about 3.5 cm in air and 20 μm in a silicon detector.

Alpha decay mechanism



Alpha particles are tightly bound systems that exist within the potential well created by the daughter nucleus.

The Q value must be **positive** for the decay to take place at all., Otherwise it is not possible for the α particle to tunnel through the barrier and escape.

Prior to emission, the α particle is considered to be confined within the potential well. It is assumed to be pre-formed inside the nucleus (not true in general). The pre-formation factor F is difficult to calculate and, normally, it is taken to be 1.

The confined alpha particle has kinetic energy and oscillates to and fro many times per second (frequency $f \sim 10^{22}$ s^{-1}).

Each time it reaches the surface, there is a small but finite probability P that the α particle can tunnel through the barrier and be emitted. This is a **quantum-mechanical** process related to the wave nature of the α particle. Once it has penetrated the barrier, the α particle is repelled away from the daughter nucleus and escapes.

A crude calculation for ^{238}U decay gives: $P \sim 10^{-39}$, which, indeed, is very small. However, the product of f and P gives a rate of escape from the nucleus of about 10^{-17} per second or about 1 chance in 3 billion years, which is comparable with the measured half-life of 4.5 billion years.

The probability P is **very** dependent on the Q value. Increasing the Q value by 1 MeV decreases the half-life by a factor of about 10 million!

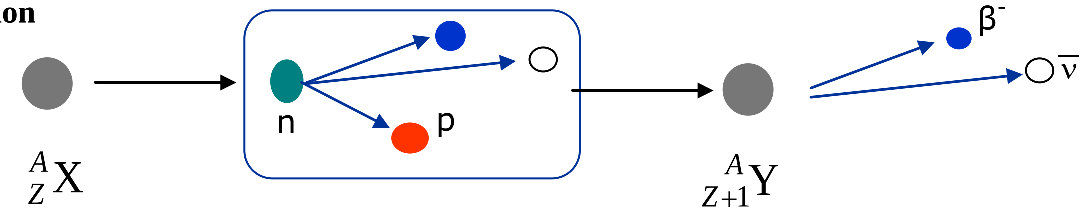
Higher-mass, α active nuclei are generally less stable, because they have higher Q values for α-decay and, hence, shorter lifetimes.

BETA DECAY

Nuclei with either too many protons or too many neutrons will undergo β decay via the weak nuclear force.

- A neutron-rich nucleus will undergo β^- emission.
- A proton-rich nucleus will either emit a β^+ particle or be transformed by capturing an atomic electron in a process called electron capture.

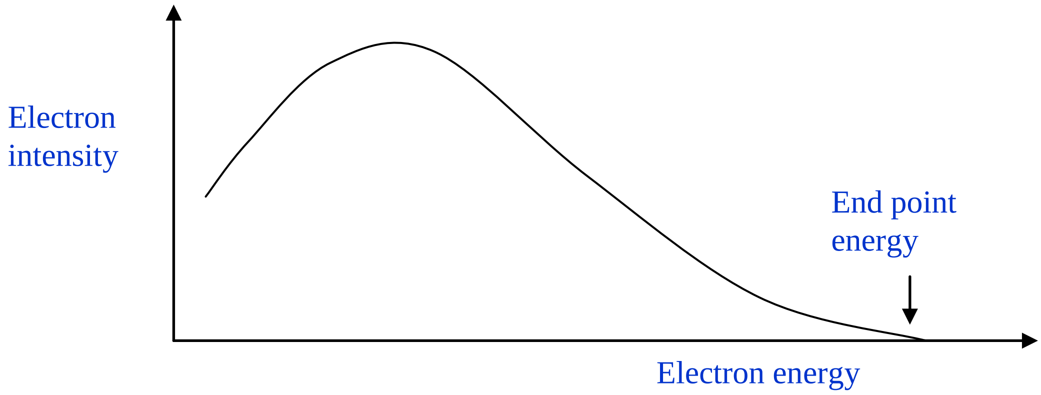
β^- emission



In β^- decay, the charge on the nucleus **increases** by one unit.

The β^- particle is an electron (mass 0.0005486 u ≡ 511 keV and charge -e). It is considered to be created at the moment of decay. A second light particle, - **antineutrino** ($\bar{\nu}$) - is also created and emitted. It has no charge and very small mass, which generally is assumed to be zero. It interacts **extremely weakly** with matter.

β^- decay is a **three-body process** - the kinetic energy released is shared between the β^- particle and the antineutrino. This results in emitted β^- particles having a **distribution of energies**, from 0 to the maximum allowed by the *Q* value (end point energy).



β^- decay involves the transformation of a neutron into a proton inside the nucleus:

$$n \rightarrow p + \beta^- + \bar{\nu} \tag{6.6}$$

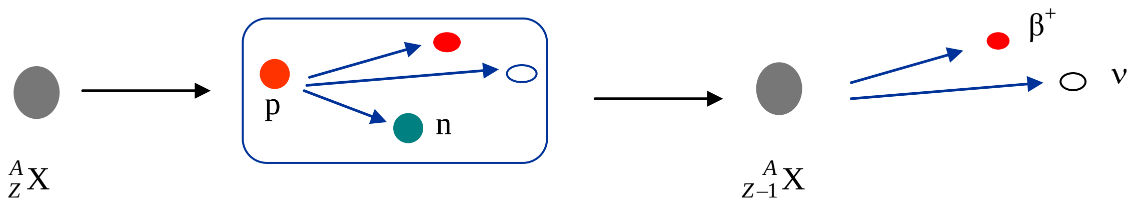
Energy must be conserved and so the transformation given in Eq. (6.6) can occur only if the daughter nucleus is **lighter** (more stable) than the parent. This means that the *Q* value:

$$Q_{\beta^-} = (m_P - m_D)c^2 \tag{6.7}$$

must be > 0. Note that the masses m_P and m_D, for the parent and daughter, respectively, are atomic not nuclear masses and include the masses of the atomic electrons.

A beta particle is much lighter than an α particle. Its speed, therefore, for a given energy is much greater and it is much more penetrating. A few mm of material will stop a 1 MeV β particle.

β^+ emission



β^+ decay changes a proton rich nucleus into a more stable isobar. The nuclear charge is decreased by one unit.

A β^+ particle is an **antielectron**, called a **positron**. It is identical to an ordinary electron except that it is positively charged. Effectively, β^+ decay converts (via the weak nuclear force) a proton into a neutron and a positron. A **neutrino** is also created in the process. Note that this is an ordinary neutrino **not** an antineutrino.

$$p \rightarrow n + \beta^{+} + \nu \qquad (6.8)$$

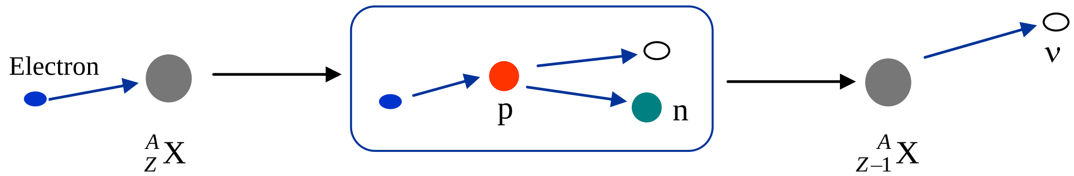
There are 3 bodies in the final state and the positrons are emitted with a continuous range of energies, as are electrons in β^{-} decay.

Using energy conservation, the β^{+} decay Q value, using atomic masses, is given by

$$Q_{\beta^{+}} = (m_{\mathrm{P}} - m_{\mathrm{D}} - 2m)c^{2} \qquad (6.9)$$

This must be greater than zero for the process to be able to occur.

Electron capture



Electron capture EC is an alternative process to β^{+} decay in that a proton is converted into a neutron. The parent absorbs an atomic electron, usually from the innermost orbit.

$$e^{-} + p \rightarrow n + \nu \qquad (6.10)$$

Energy conservation gives the Q value as

$$Q_{\mathrm{EC}} = (m_{\mathrm{P}} - m_{\mathrm{D}})c^{2} \qquad (6.11)$$

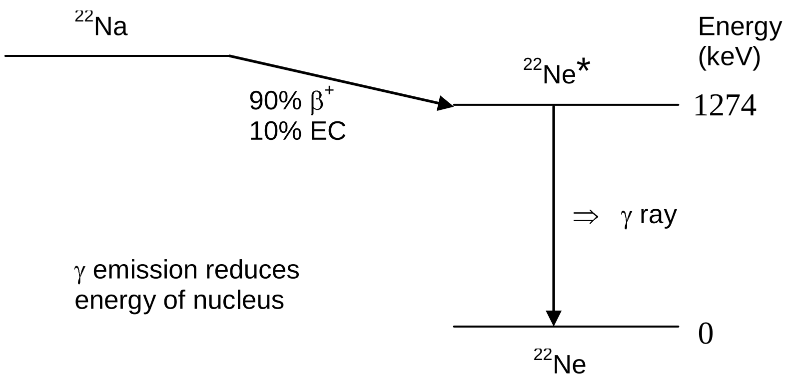
Note that Q_{EC} is greater than $Q_{\beta^{+}}$ by $2mc^{2}$. In electron capture, the mass of an atomic electron is converted into energy, whereas, in β^{+} decay, energy is required to create a positron. This means that EC can occur when β^{+} decay cannot. No β particle is emitted in electron capture and so, except for a very small amount of recoil energy of the daughter nucleus, the energy released escapes undetected.

GAMMA EMISSION AND INTERNAL CONVERSION

γ emission

A nucleus in an excited state (often after a decay) generally decays rapidly to the ground state by emitting γ rays.

Example:



Gamma-ray energies E_γ typically, are in the range 0.1 to 10 MeV and can be determined very accurately with a modern detector. E_γ is characteristic of the emitting nucleus and are widely used to identify radioactive nuclei.

Gamma rays of about an MeV do not interact strongly in matter and will penetrate many cm of moderate-density material.

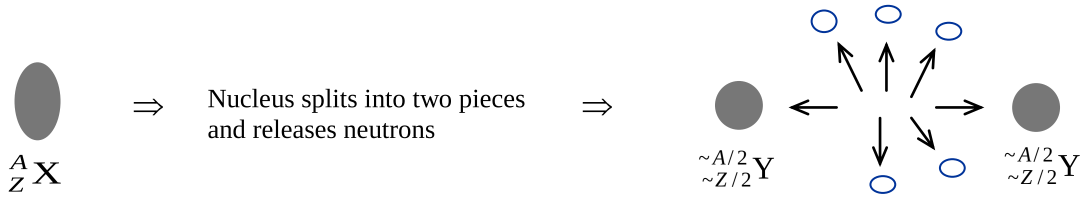
Internal conversion

This is an alternative to γ emission whereby an excited nucleus de-excites by ejecting an electron from an atomic orbit.

Both γ emission and internal conversion are due to the action of the electromagnetic force.

SPONTANEOUS FISSION AND NEUTRON EMISSION

Spontaneous fission



In spontaneous fission a nucleus breaks up into two roughly equal mass fragments. The process normally is restricted to heavy nuclei for which *BE/A* for the fragments > *BE/A* for the initial nucleus.

Fission also can be **induced** by a nuclear reaction, e.g. neutron capture.

Spontaneous fission occurs only in very heavy nuclei, such as ^{252}Cf, when the *Q* value is large enough to overcome the energy needed to **deform** the parent into two separate pieces.

Fission fragments are very **neutron rich**. Also, several (2 to 5) neutrons are emitted during fission. These are **prompt neutrons**. Their energies cover a range of ~2 MeV.

Neutrons are **uncharged** and deposit their energy in matter via nuclear interactions. This means that their interaction probability generally is small and their range is **not** well defined. Neutrons of several MeV will penetrate many 10s of cm of moderately dense materials, such as concrete.

Delayed neutrons

Neutron-rich fission products decay by β^- emission. Often, they follow a **chain** of decays to stability.

Occasionally (in 1 to 2% of cases), a daughter is formed in an excited state that can decay by **neutron emission**. Compared with β^- decay, neutron decay is **very rapid** and if there is a preceding β decay, the neutron emission is delayed – hence, the term: **delayed neutrons**.

Example: