Solutions to Problems

Problem 10.1

Adding 27.1 cm to the data set increases the mean to 24.4875 cm. This is 0.7%. $< L >^2$ becomes 599.6377 cm^2. $< L^2 >$ is 600.105 and thus $V =< L^2 > - < L >^2=$ 0.4673 giving $\sigma = 0.684$, an increase of a factor of five.

Problem 10.2

The variance is given by equation 10.7 and with the Gaussian normalised,

$$V = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x- < x >)^2 P(x)\mathrm{d}x$$

with

$$P(x) = e^{-(x-<x>)^2/2\sigma^2},$$

since x_0 in equation 10.8 equals the mean $< x >$. Changing the variable from x to $y = x- < x >$

$$V = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} y^2 \mathrm{exp}(-y^2/2\sigma^2)\mathrm{d}y,$$

and from a similar definite integral in Appendix A the integral in the above expression for V can be deduced to equal $\sigma^2\sqrt{\sigma^2 2\pi}$, giving $V = \sigma^2$.

Problem 10.3

$$\sum_{n=0}^{n=\infty} P(n) = e^{-<n>} \sum_{n=0}^{n=\infty} \frac{< n >^n}{n!}$$

From equation 6.4

$$e^{<n>} = \sum_{n=0}^{n=\infty} \frac{< n >^n}{n!},$$

hence

$$\sum_{n=0}^{n=\infty} P(n) = 1.$$

Problem 10.4

The mean number of electrons produced in the detecting equipment by a burst is $< n >= 0.012$. The probability of detecting a burst is then the probability that a burst results in at least one electron in the equipment. This is unity less the probability that no electrons are produced, and the probability of detecting a burst is

$$1 - e^{-<n>} \frac{< n >^0}{0!} = 1 - e^{-<n>} = 0.012.$$

Problem 10.5

The current $I = V/R$ and

$$\Delta I = \frac{\partial I}{\partial V}\Delta V + \frac{\partial I}{\partial R}\Delta R = \frac{1}{R}\Delta V - \frac{V}{R^2}\Delta R.$$

hence

$$\sigma_I^2 = \frac{1}{R^2}\sigma_V^2 + \frac{V^2}{R^4}\sigma_R^2 = \frac{1}{100}(0.2)^2 + \frac{144}{10000}(0.2)^2.$$

This gives σ_I =0.031 A.

Problem 10.6

The mean value of the measured resistances is $< R >= 7.3714\ \Omega$. The mean value of the temperatures is $< T >= 340$ K. The mean value of the squares of the temperatures, $< T^2 >$, is 117200 K^2 and the mean of the temperatures squared, $< T >^2$, is 115600 K^2.The mean value of the products of R and T, $< RT >$, is 2522.571 Ω K and the product $< R >< T >$ is 2506.276 Ω K. The slope b of the best fit straight line is, from equation 10.21,

$$b = \frac{< RT > - < R >< T >}{< T^2 > - < T >^2} = 0.0102\ \Omega\ \mathrm{K}^{-1}.$$

Problem 10.7

Substituting $x =< x >$ into equation 10.21 for the best fit line,

$$y = \frac{< x^2 >< y > - < x >< xy >}{< x^2 > - < x >^2} + \frac{< xy >< x > - < x >^2 < y >}{< x^2 > - < x >^2}$$

$$= \frac{< x^2 >< y > - < x >^2 < y >}{< x^2 > - < x >^2} = \frac{< y > (< x^2 > - < x >^2)}{< x^2 > - < x >^2} =< y > .$$