Solutions to Problems

Problem 2.2

For $f = a + bx$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{f(x + \mathrm{d}x) - f(x)}{\mathrm{d}x}$$

$$= \frac{a + b(x + \mathrm{d}x) - a - bx)}{\mathrm{d}x} = b\frac{\mathrm{d}x}{\mathrm{d}x} = b.$$

Problem 2.3

For $f = a + bx + cx^2$

$$\frac{f(x + \mathrm{d}x) - f(x)}{\mathrm{d}x} = \frac{a + b(x + \mathrm{d}x) + c(x + \mathrm{d}x)^2 - a - bx - cx^2}{\mathrm{d}x}$$

$$= \frac{b\mathrm{d}x + 2cx\mathrm{d}x + c\mathrm{d}x^2}{\mathrm{d}x} = b + 2cx.$$

Problem 2.4

For $f = (x^2 + \ln x)^2$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = 2(x^2 + \ln x) \times \frac{\mathrm{d}}{\mathrm{d}x}(x^2 + \ln x)$$

$$= 2(x^2 + \ln x) \times \left(2x + \frac{1}{x}\right)$$

$$= 4x^3 + 2x + 4x\ln x + \frac{2\ln x}{x}.$$

Problem 2.5

For $f = (x^2 + \ln x)^2$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = 4x^3 + 2x + 4x\ln x + \frac{2\ln x}{x}.$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 12x^2 + 2 + 4\ln x + 4 + 2\left(\frac{x/x - \ln x}{x^2}\right)$$

$$= 12x^2 + 6 + 4\ln x + \frac{2}{x^2} - \frac{2\ln x}{x^2}.$$

Problem 2.6

The values of x where $f = 2x/(3 + x^2)$ has maxima or minima are values where the first differential of f is zero.

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{2(3 + x^2) - 4x^2}{(3 + x^2)^2} = 0.$$

Hence the numerator $6 - 2x^2 = 0$ and $x = \pm\sqrt{3}$. There is no need in this case to examine the sign of the second differential at $x = \pm\sqrt{3}$ to see which corresponds to the maximum and which the minimum. The maximum value of the function is $\sqrt{3}/2$ and the minimum $-\sqrt{3}/2$.