Solutions to Problems

Problem 3.1

$$\int \left(3x^2 - 3x + 8 - \frac{1}{x}\right) \mathrm{d}x = x^3 - \frac{3}{2}x^2 + 8x - \ln x + C,$$

where C is a constant.

$$\int_1^4 \left(3x^2 - 3x + 8 - \frac{1}{x}\right) \mathrm{d}x = \left(x^3 - \frac{3}{2}x^2 + 8x - \ln x\right)_1^4$$

$$= \left(4^3 - 24 + 32 - \ln 4\right) - \left(1 - \frac{3}{2} + 8 - \ln 1\right),$$

and integral one has the value 64.5 -ln 4.

$$\int_4^8 \left(3x^2 - 3x + 8 - \frac{1}{x}\right) \mathrm{d}x = 8^3 - 96 + 64 - \ln 8 - 4^3 + 24 - 32 + \ln 4,$$

and integral two has the value 408- ln 8 + ln 4.

$$\int_1^8 \left(3x^2 - 3x + 8 - \frac{1}{x}\right) \mathrm{d}x = 8^3 - 96 + 64 - \ln 8 - 1 + \frac{3}{2} - 8 + \ln 1,$$

and integral three has the value 472.5- ln8 which equals the sum of the first two.

Problem 3.2

For $\int x^2 e^{2x} \mathrm{d}x$, in the notation of equation 3.11 put

$$f(x) = x^2 \text{ and } \frac{\mathrm{d}g(x)}{\mathrm{d}x} = e^{2x}.$$

Then

$$\int x^2 e^{2x} \mathrm{d}x = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}\int e^{2x} 2x \mathrm{d}x = \frac{1}{2}x^2 e^{2x} - \int x e^{2x} \mathrm{d}x.$$

Now use equation 3.11 again for the last integral with

$$f(x) = x \text{ and } \frac{\mathrm{d}g(x)}{\mathrm{d}x} = e^{2x}$$

to give

$$\int x e^{2x} \mathrm{d}x = \frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x} \mathrm{d}x = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}$$

and

$$\int x^2 e^{2x} \mathrm{d}x = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x}.$$

Problem 3.3 Since the limits on the y integral depend on x do the y integral first.

$$\int_{x=0}^{x=x} \int_{y=0}^{y=x+1} (4xy + 3y^2)\mathrm{d}x\mathrm{d}y = \int_0^x \left(2xy^2 + y^3\right)_0^{x+1} \mathrm{d}x$$

$$= \int_0^x (x^2 + 2x + 1)(3x + 1)\mathrm{d}x = \int_0^x (3x^3 + 7x^2 + 5x + 1)\mathrm{d}x$$

$$= \frac{3}{4}x^4 + \frac{7}{3}x^3 + \frac{5}{2}x^2 + x.$$