

## Solutions to Problems

### Problem 5.1

$$\sin 210^\circ = \sin(180^\circ + 30^\circ) = \cos 180^\circ \sin 30^\circ = -0.5.$$

$$\cos 120^\circ = \cos(90^\circ + 30^\circ) = -\sin 30^\circ = -0.5.$$

$$\tan 330^\circ = \frac{\sin 330^\circ}{\cos 330^\circ} = \frac{\sin(360^\circ - 30^\circ)}{\cos(360^\circ - 30^\circ)} = -\frac{\sin 30^\circ}{\cos 30^\circ} = -\frac{1}{\sqrt{3}}.$$

### Problem 5.2

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\sin(\theta/2 + \theta/2)}{\cos(\theta/2 + \theta/2)} \\ &= \frac{2 \sin(\theta/2) \cos(\theta/2)}{\cos^2(\theta/2) - \sin^2(\theta/2)} = \frac{2 \tan(\theta/2)}{1 - \tan^2(\theta/2)}.\end{aligned}$$

### Problem 5.3

In Figure B.1,  $a = d + f$  and

$$\frac{d + f}{\sin \alpha} = \frac{b}{\sin \beta}. \quad (B.1)$$

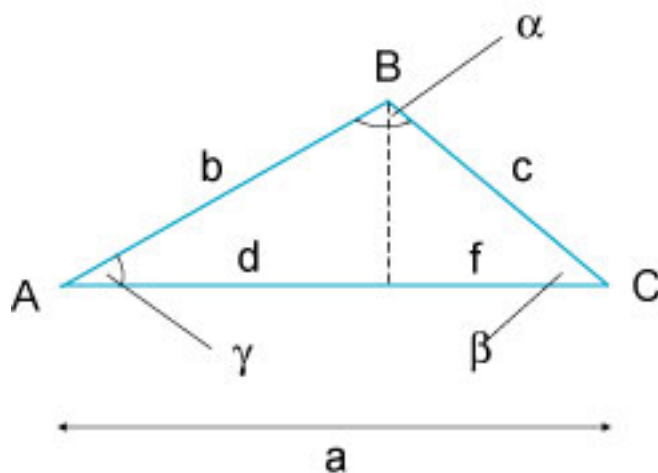


Figure B.1

From the right-angled triangle formed by dropping a line from the point B to meet the line AC at right angles,  $d = b \cos \gamma$ , and  $f = c \cos \beta$ . Substitution in equation B.1 gives

$$\frac{b \cos \gamma + c \cos \beta}{\sin \alpha} = \frac{b}{\sin \beta}$$

and

$$\begin{aligned} \sin \alpha &= \frac{\sin \beta (b \cos \gamma + c \cos \beta)}{b} \\ &= \sin \beta \cos \gamma + \frac{c}{b} \sin \beta \cos \beta \\ &= \sin \beta \cos \gamma + \left( \frac{\sin \gamma}{\sin \beta} \right) \sin \beta \cos \beta \\ &= \sin \beta \cos \gamma + \sin \gamma \cos \beta. \end{aligned}$$

But  $\beta + \gamma = \pi - \alpha$  and hence  $\sin(\beta + \gamma) = \sin \alpha$  and finally

$$\sin(\beta + \gamma) = \sin \beta \cos \gamma + \sin \gamma \cos \beta.$$

#### Problem 5.4

$$\frac{d}{d\theta} \left( \frac{\sin \theta \cos \theta}{1 + \sin \theta} \right) = \frac{(1 + \sin \theta)(\cos^2 \theta - \sin^2 \theta) - \sin \theta \cos^2 \theta}{(1 + \sin \theta)^2},$$

from equation 2.11,

$$\begin{aligned} &= \frac{\cos^2 \theta - \sin^2 \theta + \sin \theta \cos^2 \theta - \sin^2 \theta}{(1 + \sin \theta)^2} = \frac{\sin \theta \cos^2 \theta}{(1 + \sin \theta)^2} \\ &= \frac{\cos^2 \theta - \sin^2 \theta + \sin \theta \cos^2 \theta - \sin \theta (\sin^2 \theta + \cos^2 \theta)}{(1 + \sin \theta)^2} \\ &= \frac{\cos^2 \theta - \sin^2 \theta + \sin \theta (\cos^2 \theta - 1)}{(1 + \sin \theta)^2} = \frac{\cos^2 \theta - \sin^2 \theta - \sin^3 \theta}{(1 + \sin \theta)^2}. \end{aligned}$$

#### Problem 5.5

$$\begin{aligned} \int \sin^2 \theta \cos^3 \theta d\theta &= \int \sin^2 \theta \cos^2 \theta d(\sin \theta) \\ &= \int \sin^2 \theta (1 - \sin^2 \theta) d(\sin \theta) = \int (\sin^2 \theta - \sin^4 \theta) d(\sin \theta) \\ &= \frac{1}{3} \sin^3 \theta - \frac{1}{5} \sin^5 \theta = \frac{1}{3} \sin^3 \theta - \frac{1}{5} \sin^3 \theta (1 - \cos^2 \theta) \\ &= \frac{2}{15} \sin^3 \theta + \frac{1}{5} \sin^3 \theta \cos^2 \theta. \end{aligned}$$

**Problem 5.6**

From equations 5.18 and 5.19

$$\begin{aligned}\cos \theta \cos \phi &= \frac{1}{4}(e^{j\theta} + e^{-j\theta})(e^{j\phi} + e^{-j\phi}) \\ &= \frac{1}{4}(e^{j(\theta+\phi)} + e^{-j(\theta+\phi)} + e^{j(\theta-\phi)} + e^{-j(\theta-\phi)}).\end{aligned}\tag{B.2}$$

$$\begin{aligned}\sin \theta \sin \phi &= -\frac{1}{4}(e^{j\theta} - e^{-j\theta})(e^{j\phi} - e^{-j\phi}) \\ &= -\frac{1}{4}(e^{j(\theta+\phi)} + e^{-j(\theta+\phi)} - e^{-j(\theta-\phi)} - e^{j(\theta-\phi)}).\end{aligned}\tag{B.3}$$

Addition of equations B.2 and B.3 gives

$$\cos \theta \cos \phi + \sin \theta \sin \phi = \cos(\theta - \phi).$$

**Problem 5.7**

$$\begin{aligned}2 \sin \frac{(\theta + \phi)}{2} \cos \frac{(\theta - \phi)}{2} \\ = \left(2 \sin \frac{\theta}{2} \cos \frac{\phi}{2} + 2 \sin \frac{\phi}{2} \cos \frac{\theta}{2}\right) \times \left(\cos \frac{\theta}{2} \cos \frac{\phi}{2} + \sin \frac{\theta}{2} \sin \frac{\phi}{2}\right).\end{aligned}$$

Multiplying out the brackets and using equation 5.7 gives

$$\begin{aligned}2 \sin \frac{(\theta + \phi)}{2} \cos \frac{(\theta - \phi)}{2} &= 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + 2 \sin \frac{\phi}{2} \cos \frac{\phi}{2} \\ &= \sin \theta + \sin \phi.\end{aligned}$$

**Problem 5.8**

$$\frac{(2 - 3j)(1 + 2j)}{4 + 3j} = \frac{8 + j}{4 + 3j} = \frac{(8 + j)(4 - 3j)}{\sqrt{4^2 + 3^2}} = \frac{7 - 4j}{5}$$

Expressing  $z$  in polar form, the modulus

$$r = \sqrt{(7/5)^2 + (4/5)^2} = \frac{1}{5}\sqrt{65}$$

and the angle  $\theta$  is given by

$$\tan \theta = -\frac{4/5}{7/5} = -4/7.$$