Solutions to Problems

Problem 6.1

$$\sum_{0}^{5} = 3\frac{(x^6-1)}{(x-1)} \quad \text{and} \quad \sum_{0}^{2} = 3\frac{(x^2-1)}{(x-1)}.$$

Hence

$$\sum_{2}^{5} = \frac{3(x^6-1)-3(x^2-1)}{(x-1)} = 3\frac{x^2(x^4-1)}{(x-1)}.$$

Problem 6.2

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

$$= \frac{1}{2j}\left(1 + jx - \frac{x^2}{2!} - \frac{jx^3}{3!} = \frac{x^4}{4!} + \frac{jx^5}{5!}.....\right)$$

$$- \frac{1}{2j}\left(1 - jx - \frac{x^2}{2!} + \frac{jx^3}{3!} + \frac{x^4}{4!} - \frac{jx^5}{5!}.....\right)$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}.....$$

Problem 6.3

Expand $(1-x)^{-1}$ using the binomial theorem with $n = -1$.

$$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2$$

$$+ \frac{(-1)(-2)(-3)}{3!}(-x)^3 + \frac{(-1)(-2)(-3)(-4)}{4!}(-x)^4...$$

$$= 1 + x + x^2 + x^3 + x^4...$$

Problem 6.4

$$\ln\frac{1+x}{1-x} = \ln(1+x) - \ln(1-x).$$

For $x < 1$ the result of Example 6.2 can be used to expand the logarithmic terms and subtracting them gives the answer.

Problem 6.5

Use Taylor's theorem to expand f and g close to zero at $x = \delta x$.

$$\frac{f(\delta x)}{g(\delta x)} = \frac{f(0) + \delta x(\mathrm{d}f/\mathrm{d}x) + ...}{g(0) + \delta x(\mathrm{d}g/\mathrm{d}x) + ...}$$

where the differentials are evaluated at $x = 0$ and higher differentials than the first can be ignored as we take the limit of $\delta x \to 0$ when

$$\frac{f(0)}{g(0)} = \frac{(\mathrm{d}f/\mathrm{d}x)_0}{(\mathrm{d}g/\mathrm{d}x)_0}.$$
