Solutions to Problems

Problem 8.1

For$f=(x+y)^3-x^2$, $\partial f/\partial x=3(x+y)^2-2x$ and $\partial^2 f/\partial x\partial y=6(x+y)$.
$\partial f/\partial y=3(x+y)^2$ and $\partial^2 f/\partial y\partial x=6(x+y)$.

Problem 8.2

For $f=\ln(x^3+y^2+z)$, $\partial f/\partial x=3x^2(x^3+y^2+z)^{-1}$ and $\partial^2 f/\partial x\partial y=-6x^2y(x^3+y^2+z)^{-2}$.

Also $e^f=x^3+y^2+z$ and thus $z=e^f-x^3-y^2$, and $\partial z/\partial y=-2y$. Reverting to the expression for f, $\partial f/\partial z=(x^3+y^2+z)^{-1}$. Hence

$$\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial z}{\partial y}\right)=-\frac{2y}{(x^3+y^2+z)}$$

and

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial z}{\partial y}\right)=6x^2y(x^3+y^2+z)^{-2},$$

which is $-\partial^2 f/\partial x\partial y$.

Problem 8.3

With the potential zero equation 8.7 becomes

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)=E\phi$$

and the variables can be separated as in Example 8.3 to give

$$-\frac{\hbar^2}{2m}\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}\right]=E.$$

Each of the terms like $-(\hbar^2/2m)(1/X)(\partial^2X/\partial x^2)$ is equal to a constant, say E_x, E_y and E_z, where $E_x+E_y+E_z=E$, and we have three separate equations like

$$\frac{\partial^2 X}{\partial x^2}=-\frac{2mE_x}{\hbar^2}X$$

to solve. Since $E_x>0$, the solution is given by equation 7.10, $X=A\sin\omega x+B\cos\omega x$ with $\omega^2=2mE_x/\hbar^2$. But X is zero at $x=0$ and $x=a$. Hence $B=0$ and $\omega a=n\pi$ where n is an integer. Substituting back in the differential equation for X gives $E_x=n^2\hbar^2\pi^2/2ma^2$. The same procedure gives similar expressions for E_y and E_z, and the lowest value of E is when $n=1$ and $E=3\pi^2\hbar^2/2ma^2$.