Solutions to Problems

Problem 9.1

In the first situation $x = 20\sin 30^\circ = 10\sqrt{3}$ cm; $y = 20\cos 30^\circ = 10$ cm, and $z = 0$. In the second, $x = 20\cos 60^\circ = 10$ cm; $y = 20\sin 60^\circ = 10\sqrt{3}$ cm, and $z = 10$ cm.

Problem 9.2

The contribution to the moment of inertia from the infinitesimal volume element at $(x,\ y,\ z)$ is its mass $\rho \mathrm{d}x\mathrm{d}y\mathrm{d}z$ times the square of its distance from the axis, $(x^2 + y^2)$. The moment of inertia is

$$I = \rho \int_{x=-a/2}^{a/2} \int_{y=-a/2}^{a/2} \int_{z=-a/2}^{a/2} (x^2 + y^2)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \rho a^2 \left[\frac{1}{3}x^3\right]_{-a/2}^{a/2} + \rho a^2 \left[\frac{1}{3}y^3\right]_{-a/2}^{a/2}$$

$$= \frac{\rho a^5}{6} = \frac{1}{6}Ma^2.$$

Problem 9.3

$$V = \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi = 2\pi \int_0^R \int_0^\pi r^2 \mathrm{d}r\mathrm{d}(\cos\theta)$$

$$= -\frac{2\pi R^3}{3}[\cos\theta]_0^\pi = \frac{4}{3}\pi R^3.$$

Problem 9.4

The difference between this volume and that of the whole sphere is that the limits on the integration over the angle θ are from θ_1 shown in Figure B.2 to π and not over the whole range of θ.

$$V = \int_{r=0}^{R} \int_{\theta=\theta_1}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi = 2\pi \int_0^R \int_0^\pi r^2 \mathrm{d}r\mathrm{d}(\cos\theta)$$

$$= -\frac{2\pi R^3}{3}[\cos\theta]_{\theta_1}^\pi = \frac{2}{3}\pi R^3(1 + \cos\theta_1) = \frac{2}{3}\pi R^3(1 + \frac{a}{R}),$$

since $\cos\theta_1 = a/R$.

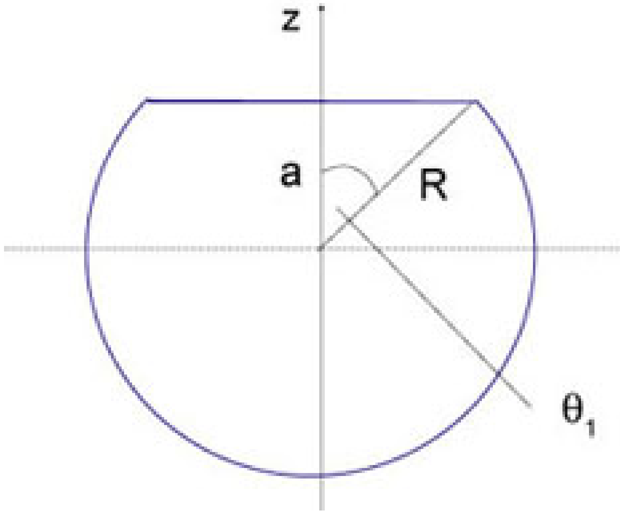

Figure B.2

Problem 9.5

From Figure B.3

$$V = \int_{r=0}^{Rz/h} \int_{\phi=0}^{2\pi} \int_{z=0}^{h} r \mathrm{d}r \mathrm{d}\phi \mathrm{d}z$$

The upper limit on r now depends on z and so do the r integral first to give

$$V = 2\pi \int_0^h \frac{1}{2} \left(\frac{R^2 z^2}{h^2} \right) \mathrm{d}z = \frac{1}{3} \pi R^2 h.$$

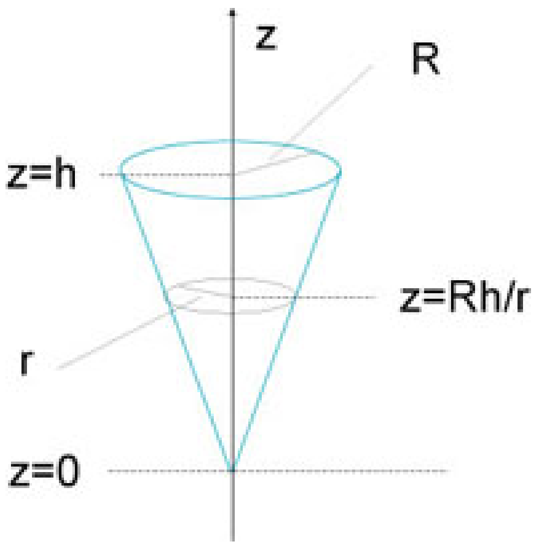

Figure B.3

Problem 9.6

With ρ the density, the moment of inertia

$$I = \rho \int_{r=0}^{R} \int_{\phi=0}^{2\pi} \int_{z-0}^{h} r^3 \mathrm{d}r \mathrm{d}\phi \mathrm{d}z = \frac{1}{4}\rho R^4 2\pi h = \frac{1}{2} M R^2.$$